

TRIGONOMETRIC FUNCTIONS

$\sin(\theta) = \frac{O}{H}$
$\cos(\theta) = \frac{A}{H}$
$\tan(\theta) = \frac{O}{A}$
$\cot(\theta) = \frac{A}{O}$
$\sec(\theta) = \frac{H}{A}$
$\csc(\theta) = \frac{H}{O}$

RECIPROCAL IDENTITIES

$\sin(\theta) = \frac{O}{H}$	$\csc(\theta) = \frac{H}{O}$	$\csc(\theta) = \frac{1}{\sin(\theta)}$
$\cos(\theta) = \frac{A}{H}$	$\sec(\theta) = \frac{H}{A}$	$\sec(\theta) = \frac{1}{\cos(\theta)}$
$\tan(\theta) = \frac{O}{A}$	$\cot(\theta) = \frac{A}{O}$	$\cot(\theta) = \frac{1}{\tan(\theta)}$

RATIO IDENTITIES

$\sin(\theta) = \frac{O}{H}$	$\csc(\theta) = \frac{H}{O}$
$\cos(\theta) = \frac{A}{H}$	$\sec(\theta) = \frac{H}{A}$
$\tan(\theta) = \frac{O}{A}$	$\cot(\theta) = \frac{A}{O}$
$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$	$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$
$\tan(\theta) = \frac{\sec(\theta)}{\csc(\theta)}$	$\cot(\theta) = \frac{\csc(\theta)}{\sec(\theta)}$

PYTHAGOREAN
IDENTITIES

$$\cos(\theta) = \frac{A}{H}$$

$$\sin(\theta) = \frac{O}{H}$$

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\frac{\sin^2(\theta)}{\cos^2(\theta)} + \frac{\cos^2(\theta)}{\cos^2(\theta)} = \frac{1}{\cos^2(\theta)}$$

$$\tan^2(\theta) + 1 = \sec^2(\theta)$$

$$\frac{\sin^2(\theta)}{\sin^2(\theta)} + \frac{\cos^2(\theta)}{\sin^2(\theta)} = \frac{1}{\sin^2(\theta)}$$

$$1 + \cot^2(\theta) = \csc^2(\theta)$$

SYMMETRY
IDENTITIES

The graph of $\sin(\theta)$ is symmetric about the origin,
therefore...

$$\sin(-\theta) = -\sin(\theta)$$

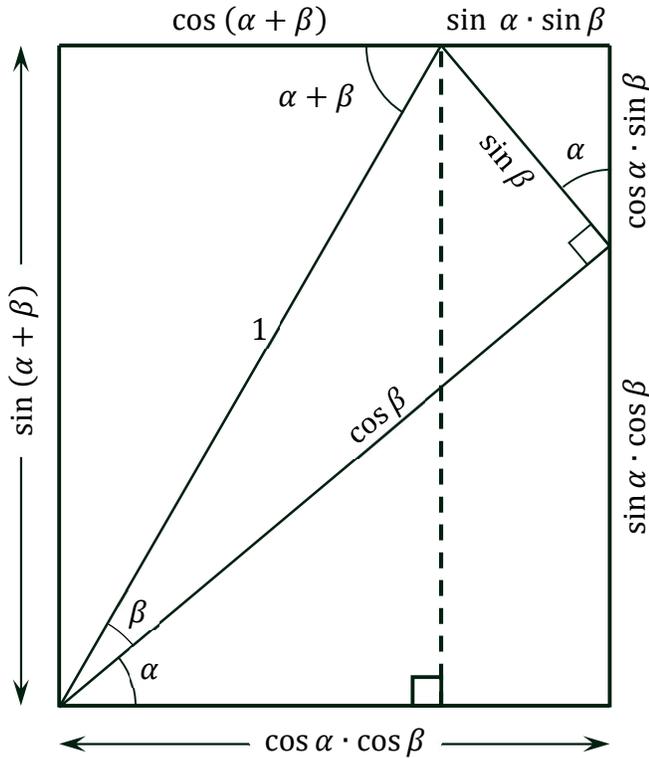
The graph of $\cos(\theta)$ is symmetric about the y-axis,
therefore...

$$\cos(-\theta) = \cos(\theta)$$

The graph of $\tan(\theta)$ is symmetric about the origin,
therefore...

$$\tan(-\theta) = -\tan(\theta)$$

SIN and COS of SUMS (and DIFFERENCES)



$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\begin{aligned} \sin(\alpha - \beta) &= \sin(\alpha + (-\beta)) \\ &= \sin \alpha \cdot \cos(-\beta) + \cos \alpha \cdot \sin(-\beta) \end{aligned}$$

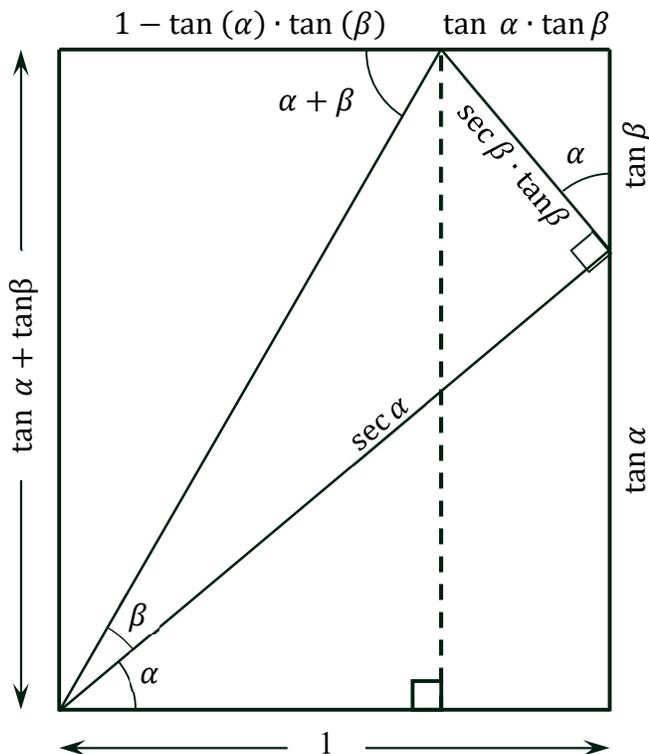
$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\begin{aligned} \cos(\alpha - \beta) &= \cos(\alpha + (-\beta)) \\ &= \cos \alpha \cdot \cos(-\beta) - \sin \alpha \cdot \sin(-\beta) \end{aligned}$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

TAN of SUMS (and DIFFERENCES)



$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta}$$

$$\tan(-\beta) = -\tan \beta$$

$$\begin{aligned} \tan(\alpha - \beta) &= \tan(\alpha + (-\beta)) \\ &= \frac{\tan \alpha + \tan(-\beta)}{1 - \tan \alpha \cdot \tan(-\beta)} \end{aligned}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \cdot \tan \beta}$$

DOUBLE ANGLE FORMULAE

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(2\theta) = \sin(\theta + \theta) = \sin \theta \cdot \cos \theta + \cos \theta \cdot \sin \theta$$

$$\sin(2\theta) = 2 \cdot \sin \theta \cdot \cos \theta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(2\theta) = \cos(\theta + \theta) = \cos \theta \cdot \cos \theta - \sin \theta \cdot \sin \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$\cos(2\theta) = (1 - \sin^2 \theta) - \sin^2 \theta = 1 - 2 \cdot \sin^2 \theta$$

$$\cos(2\theta) = \cos^2 \theta - (1 - \cos^2 \theta) = 2 \cdot \cos^2 \theta - 1$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta}$$

$$\tan(2\theta) = \tan(\theta + \theta) = \frac{\tan \theta + \tan \theta}{1 - \tan \theta \cdot \tan \theta}$$

$$\tan(2\theta) = \frac{2 \cdot \tan \theta}{1 - \tan^2 \theta} \cdot \left(\frac{\cos^2 \theta}{\cos^2 \theta} \right) =$$

$$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)} = \frac{2 \cdot \sin \theta \cdot \cos \theta}{\cos^2 \theta - \sin^2 \theta}$$

HALF ANGLE FORMULAE

$$\cos(2\theta) = 1 - 2 \cdot \sin^2\theta$$

$$\cos(2\theta) = 2 \cdot \cos^2\theta - 1$$

$$2 \cdot \sin^2\theta = 1 - \cos(2\theta)$$

$$2 \cdot \cos^2\theta = 1 + \cos(2\theta)$$

$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$

$$\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$$

$$\sin \theta = \pm \sqrt{\frac{1 - \cos(2\theta)}{2}}$$

$$\cos \theta = \pm \sqrt{\frac{1 + \cos(2\theta)}{2}}$$

$$\frac{\theta}{2} = \theta$$

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos(\theta)}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos(\theta)}{2}}$$

$$\tan \frac{\theta}{2} = \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} = \frac{\pm \sqrt{\frac{1 - \cos(\theta)}{2}}}{\pm \sqrt{\frac{1 + \cos(\theta)}{2}}}$$

$$\tan \frac{\theta}{2} = \frac{\sqrt{1 - \cos(\theta)}}{\sqrt{1 + \cos(\theta)}} \cdot \frac{\sqrt{1 + \cos(\theta)}}{\sqrt{1 + \cos(\theta)}} = \frac{\sqrt{1 - \cos^2(\theta)}}{1 + \cos(\theta)}$$

$$= \frac{\sin(\theta)}{1 + \cos(\theta)}$$

$$\tan \frac{\theta}{2} = \frac{\sqrt{1 - \cos(\theta)}}{\sqrt{1 + \cos(\theta)}} \cdot \frac{\sqrt{1 - \cos(\theta)}}{\sqrt{1 - \cos(\theta)}} = \frac{1 - \cos(\theta)}{\sqrt{1 - \cos^2(\theta)}}$$

$$= \frac{1 - \cos(\theta)}{\sin(\theta)}$$

$$\tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos(\theta)}{1 + \cos(\theta)}}$$